**viscoelastic modeling of porcine ligaments**

**Bruno Mello Silveira**

Stephanie Aguiar Salles de Barros

Departamento de Engenharia Mecânica - PPEMM - CEFET/RJ; Av. Maracanã, 229 – RJ – Brazil. bruno.silveira@aluno.cefet-rj.br, stephanie.barros@aluno.cefet-rj.br

Rodrigo Ribeiro Pinho Rodarte

Programa de Pós-graduação em Engenharia Mecânica e Tecnologia de Materiais - PPEMM - CEFET/RJ; Av. Maracanã, 229 – RJ – Brazil. Instituto Nacional de Traumatologia e Ortopedia – INTO -Av. Brazil, 500, RJ, Brazil. rodrigo.rodarte@aluno.cefet-rj.br

**Paulo Pedro Kenedi**

Programa de Pós-graduação em Engenharia Mecânica e Tecnologia de Materiais - PPEMM - CEFET/RJ; Av. Maracanã, 229 – RJ – Brazil. Departamento de Engenharia Mecânica – CEFET/RJ - Av. Maracanã, 229 – RJ – Brazil. paulo.kenedi@cefet-rj.br

**Abstract.** Viscoelastic quasi-linear analytical models, as Fung, was implemented through the utilization of experimental results obtained from several porcine ligaments as: lateral collateral ligament (LCL), anterior cruciate ligament (ACL), posterior cruciate ligament (PCL) and medial collateral ligament (MCL). To implement quasi-linear viscoelastic models for soft tissues, as the Fung one, was necessary the utilization of a programming language, as C Sharp, and Object-oriented programming to deal with the model’s mathematical demands, as the convolution calculations. Moreover, those technologies allow to reduce the code execution time which was one of the main problems. Despite this benefit, was necessary to implement the numerical methods used in process. The models’ results show the stress evolution in relaxation tests. Although, the preliminary results show a good correlation between experimental and analytical models, showing a noticeable change in ligaments stiffness after the experimental implementation of relaxation tests.

**Keywords:** knee ligaments, analytic model, viscoelasticity, Fung

1. Introduction

The knee is one of the most complex joints in the body being subjected to different efforts. Studying and correctly describing the mechanical behavior of the knee ligaments is extremely important. Thus, a huge number of researches were published trying to macroscopically analyze these tendons through different viscoelastic mechanical models. (Rossetto, 2009) shows that this knowledge is important for better analyzes to be made to determine physical training, such as in cases of therapy for tendinopathies. (Bernardes et. al, 2005) sought to determine the biomechanical parameters for modeling the human knee joint through extensive exercises, together with images obtained by videofluoroscope, where viscoelasticity plays an important role.

Viscoelasticity is understood as the property of materials that present viscous and elastic behavior at the same time, being a concept widely used in various sectors of the industry. The simplest viscoelastic model is one that considers linear functions, where the creep compliance and stress relaxation functions depending only on time, it is commonly used for metals. (Tareco, 2014) uses Maxwell and Kelvin linear models to model a steel-concrete structure, analyzing the relaxation and creep compliance just for the concrete in the mixed structure response. Moreover, as presented by (Queiroz, 2008), viscoelastic materials are also used to attenuate vibrations and noise in structures, having application in both the automotive and aerospace sectors.

Since linear models do not represent the real behavior, quasi-linear or non-linear models are used for a better approach, being the first the focuses of this study. Moreover, those methods are frequently applied with computational resources.

The viscoelastic quasi-linear model, proposed by Fung (Fung, 1993), is commonly used in soft tissue research since it describes the behavior nearly the reality.

Moreover, to deal with complexes integrations and derivatives and functions that do not have analytical solutions, a software must be used

The aim of this paper is to explain how to implement numerically the Fung’s quasi-linear viscoelasticity model, showing the flowcharts and the results, and comparing with the experimental analysis. To obtain the results, it was developed an API using C#,

“an object-oriented, component-oriented programming language”, according with (Wagner et al, 2021).

to deal with model’s mathematical demands, as integral and derivative calculations

1. Fung’s quasi-linear viscoelastic model

The quasi-linear viscoelastic model, proposed by (Fung, 1993), applies the non-linearity stress-strain relation expressing the stress in two parts: the reduced relaxation function, which depends only on time, and elastic response, which depends on strain. To improve the stress calculations, the elastic response can be expressed depending only on time, because, in that research, the strain is considered depending on this, as will be shown below. This model is commonly used for soft tissue, as it can represent the tissue with good approximation. The constants needed for the equations is obtained experimentally, however, like any model, the Fung’s model has limitations, since for distinct relaxations and strain levels, different values for those constants are found. Moreover, two considerations were made: consider and disregard ramp time. While considering, is possible to calculate the variables A and B, shown below in elastic response equation, with those, multiple relaxations can be assumed. In that research, is assumed just two relaxations. In table 1, the Fung’s model constants for each tissue are presented with their value for the first and second relaxation, these were obtained by the research team experimentally, as mentioned above.

Table 1. Fung’s model constant for first and second relaxation used for each tissue.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Material Constants** | **ACL** | | **PCL** | | **LCL** | | **MCL** | |
| **First** | **Second** | **First** | **Second** | **First** | **Second** | **First** | **Second** |
| **G1** | 0.27 | 0.063 | 0.03 | 0.028 | 0.10 | 0.11 | 0.018 | 0.02 |
| **G2** | 0.18 | 0.098 | 0.05 | 0.046 | 0.18 | 0.17 | 0.032 | 0.03 |
| **G3** | 0.56 | 0.24 | 0.11 | 0.077 | 0.32 | 0.24 | 0.059 | 0.04 |
| **G∞** | 1.81 | 1.64 | 1.43 | 1.387 | 3.79 | 3.55 | 0.86 | 0.81 |
| **τ1**, s | 19.33 | 2.35 | 0.97 | 1.67 | 1.02 | 1.88 | 0.59 | 0.96 |
| **τ2**, s | 19.6 | 19.86 | 6.92 | 13.91 | 7.52 | 15.63 | 4.73 | 9.38 |
| **τ3**, s | 370.8 | 213.64 | 53.18 | 134.91 | 53.38 | 150.0 | 32.29 | 95.36 |
| **A, MPa** | 112.96 | 54.88 | 1.86 | 1.77 | 3.94 | 7.27 | 9.47 | 11.58 |
| **B** | 0.34 | 0.86 | 10.15 | 12.79 | 1.19 | 6.17 | 1.39 | 1.77 |

* 1. Mathematical equation

(Fung, 1993) propose equations for elastic response, reduced relaxation function and stress considering one relaxation, although, as two relaxations will be considered, it is necessary to reformulate these equations. Moreover, each parameter will be expressed differently when considering and disregarding ramp time, except for reduced relaxation function.

* + 1. Strain

The equations used to describe the strain were developed to represent the experiments. When considering ramp time, the strain behavior is expressed in equation 1, that when the strain maintains at the maximum value , it represents the relaxations and when stays at the minimum value , represents the recovery, that behavior also is observed in (Duenwald, et al., 2009). When disregard ramp time, the equation 2 is used, where is considered a constant strain while all experiment.

|  |  |
| --- | --- |
| , | (1) |

where, the parameters and represent, respectively, ramp time and strain rate applied in experiment, with used when strain increase and , when decrease. Furthermore, the parameters , , and are the limit time for each equation, indicating when the strain behavior changes.

|  |  |
| --- | --- |
| , | (2) |

where, represents the constant stain applied in experiment.

(a) (b)

Figure 1. Strain per time when (a) considering and (b) disregarding ramp time.

With that, is possible to calculate the derivative that will be used in the stress calculations step. The equation 3 and 4 are the derivative in time of, respectively, equations 1 and 2.

|  |  |
| --- | --- |
| , | (3) |
| . | (4) |

(a) (b)

Figure 2. Strain derivative in time per time when (a) considering and (b) disregarding ramp time.

* + 1. Elastic response

The elastic response corresponds the soft tissue elastic part. As mentioned previously, two equations will be used to describe the elastic response. When considering ramp time, an exponential approximation has been chosen like used in research (Abramowitch, 2004).

|  |  |
| --- | --- |
| , | (5) |

where constants A, in Pa (Pascal), and B, dimensionless, are material constants and represents, respectively, elastic stress constant and elastic power constant. Moreover, as shown previously, the equation 5 is rewritten with the aim to elastic response only depends in time.

|  |  |
| --- | --- |
| , | (6) |

When disregarding ramp time, the elastic response is considered constant for all time domain, because the strain is constant, and that parameter depends on strain.

|  |  |
| --- | --- |
| , | (7) |

where, is the initial stress applied in experiment.

(a) (b)

Figure 3. Elastic response per time when (a) considering and (b) disregarding ramp time.

(a) (b)

Figure 4. Elastic response per strain when (a) considering and (b) disregarding ramp time.

As made for strain, the derivative for elastic response must be calculated because it will be used in equations for describe the stress.

The derivative in time and in strain for equation 6:

|  |  |
| --- | --- |
| , | (8) |
| . | (9) |
| The derivative in time and in strain for equation 7:  . | (10) |

(a) (b)

Figure 5. Elastic response derivative (a) in time per time and (b) in strain per strain when considering ramp time.

* + 1. Reduced relaxation funcion

The reduced relaxation function represents the viscous portion and occurs for all time domain begging at 1, **g**(0) = 1. According with (Fung, 1993), it can be described in two ways. The first, equation 9, also called the simplified reduced relaxation function, is written as the Prony Series, but using only three elements in the sum, while according with (Babaei et al, 2015), those are sufficient for a good approximation, and (Funk et al., 2000) more than three elements do not result in significant gain. The second, equation 10, was developed from Kelvin model, standard linear solid (Fung, 1993), and uses integrals that only have numerical solutions. Moreover, both equations were implemented and tested but only the first was used, as constants are easier to be calculated experimentally.

|  |  |
| --- | --- |
| , | (9) |

where and are material dimensionless constants called relaxation modulus and represents the amplitude of the stress curve in relaxation, and is the relaxation time in seconds, also a material constant.

|  |  |
| --- | --- |
| , | (10) |

where C, and are material constants and represents, respectively, a dimensionless relaxation constant, fast and slow relaxation times in second. To improve the numerical implementation, that equation was rewritten as shown below.

Based on material properties and the constants definition, can be assumed that , so, , therefore, could be rewritten like:

|  |  |
| --- | --- |
| , |  |

Then:

|  |  |
| --- | --- |
| , | (11) |

Applying equation 11 in 10, is found:

|  |  |
| --- | --- |
| , | (12) |

(a) (b)

Figure 6. (a) Reduced Relaxation Function and (b) Simplified Reduced Relaxation Function per time

Also calculating the derivative in time for each equation for reduced relaxation function.

Deriving (9):

|  |  |
| --- | --- |
| , | (13) |

Deriving (12):

|  |  |
| --- | --- |
| , | (14) |
| , |  |

Applying the definition of calculus to the derivative of a definite integral:

|  |  |
| --- | --- |
| , |  |

where , and b.

|  |  |
| --- | --- |
| , | (15) |

Applying (15) in (14):

|  |  |
| --- | --- |
| , | (16) |

(a) (b)

Figure 7. Derivative in time of (a) Reduced Relaxation Function and (b) Simplified Reduced Relaxation Function per time

* + 1. Stress

(Fung, 1993) shows three equivalent equations to calculate the stress:

|  |  |
| --- | --- |
| , | (17) |
| , | (18) |
| , | (19) |

As mentioned above, the elastic response and reduced relaxation function can be expressed only depending on time, so the partial derivative can be changed by total derivative. Moreover, and .

|  |  |
| --- | --- |
| , | (20) |
| , | (21) |
| , | (22) |

(a) (b) (c)

Figure 8. Stress per time when considering ramp time using equation (a) 20, (b) 21 and (c) 22.

While considering ramp time, equations 20 and 22 return satisfactory results, however the results obtained with equation 21 diverged from the others, like shown in Fig. 8. Disregarding ramp time, the elastic response is constant and its derivative is zero for all time domain, as shown previously. Thus, the equation 20 cannot be used, because it always returns zero since, and equations 21 and 22 can be rewritten.

Rewriting (21):

|  |  |
| --- | --- |
| , |  |
| , |  |
| , |  |
| , |  |
| . | (23) |

Rewriting (22):

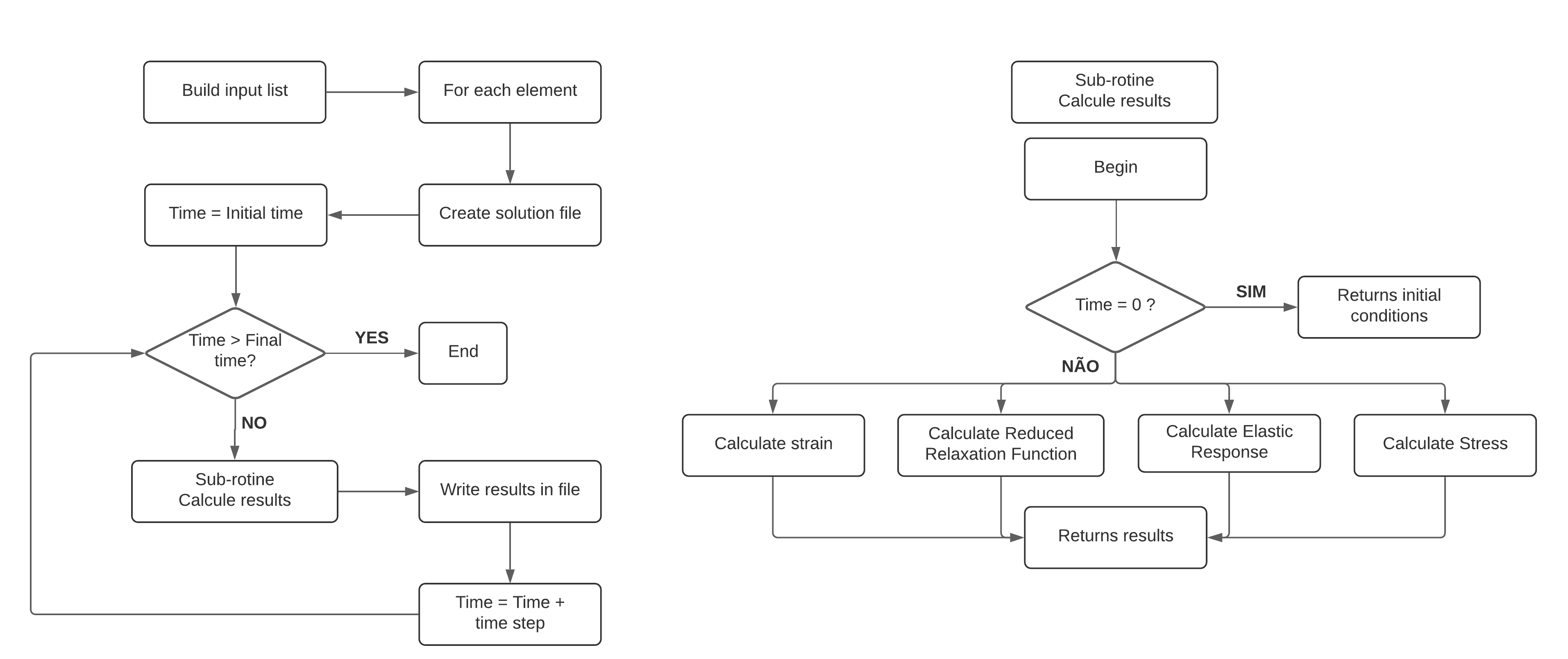
|  |  |
| --- | --- |
| , |  |
| , |  |
| . | (24) |

As presented, (23) and (24) are equals, so when disregarding ramp time, a unique equation must be used.

Figure 9. Stress per time when disregarding ramp time.

1. numerical implementation

The numerical implementation of Fung’s Model was developed by two steps: creating a class that represents the model and contains the equations for each parameter and its derivative in time; and creating a class to orchestrate the operation. Also was created an artificial frontier in the code that separate the operation and the model, being created specific contracts for each one. This was done based on Single-Responsibility Principle that gets easier to implement resources, prevents unexpected side-effects and improves maintainability. It is noteworthy that the execution time was minutes and, in worst cases, hours, because the equations used to calculate the stress are not optimized for numeric applications. To improve that was used the class Task, a native resource from C#, with the aim to let some steps asynchronous, executing multiple tasks together and reducing the execution time. It was used in both classes mentioned, in first, when calculating the results, and, in second, when iterating the input list, reducing that time to seconds, in worst case.



(a) (b)

Figure 10. Flowchart for (a) main operation and (b) sub-routine “Calculate Results”.

The class that represents the model also contains a method, represented in Fig. 10 as sub-routine “Calculate Results”, that calculate in parallel all results necessaries - strain, elastic response, reduced relaxation function and stress – as shown in Fig. 10.b, and returns those values in an object. The orchestrator, as called, is responsible to orchestrate the operation, executing each step shown on Fig. 10.a, furthermore, previously the request data is validated to certain if it is valid and any error will be thrown during code execution.

What is more, it was necessary to implement numerical methods to deal with integrations and derivatives present in stress and reduced relaxation function equations. For the integrals, the Composite Simpson's Rule, equation 25, (Regra de Simpson, 2021) (Regras Compostas, 2021) to grant greater precision, since this was the best solution found when compared with other methods using a same time step. For the derivatives, the Symmetric Derivative, equation 26, (Da Cruz, 2012) was used since it gives the precision necessary while calculating the parameters.

|  |  |
| --- | --- |
| , | (25) |

where f(x) is an integrable function, a and b are the limits of integration, x is a differential of the variable x, and N is the number of subdivisions.

|  |  |
| --- | --- |
| , | (26) |

where f(x) is a differentiable function and x is a differential of the variable x.

1. RESULTS AND CONCLUSIONS

ESCREVER UM RESUMO DO QUE ACONTECEU EXPLICANDO O QUE FOI ENCONTRADO E ACONTECEU.

1. References

Silveira, B. M. 2019. “SoftTissue”. Available in: https://github.com/M3110/SoftTissue. Accessed on June 01, 2021.

Fung, Y. 1993. “Biomechanics: Mechanical Properties of Living Tissues”. Springer, New York, University of Michigan.

Duenwald, S. E.; Jr., R. V.; Lakes, R. S., 2009. “Viscoelastic Relaxation and Recovery of Tendon”. University of Wisconsin-Madison. Madison, USA.

Abramowitch, Steven D. e WOO, Savio L.-Y., 2004. “An Improved Method to Analyze the Stress Relaxation of Ligaments Following a Finite Ramp Time Based on the Quasi-Linear Viscoelastic Theory”. JOURNAL OF BIOMECHANICAL ENGINEERING. ASME. Vol. 126. P. 92-97. DOI: 10.1115/1.1645528.

Funk, J. R., Hall, G. W., Crandall, J. R., Pilkey, W. D., 2000. “Linear and quasi-linear viscoelastic characterization of ankle ligaments”. Journal of Biomechanical Engineering, Vol. 122, No. 1, p. 15-22. DOI:10.1115/1.429623

Regra de Simpson. Instituto Superior Técnico de Lisboa. Available in: https://www.math.tecnico.ulisboa.pt/ ~calves/courses/integra/capiii33.html#:~:text=Regra%20de%20Simpson%20aplicada%20a%20dois%20sub%2Dintervalos.&text=Assim%2C%20podemos%20considerar%20tr%C3%AAs%20n%C3%B3s,cada%20um%20destes%20sub%2Dintervalos. Accessed on: May 18, 2021.

Regras Compostas. Universidade Federal do Rio Grande do Sul. Available in: https://www.ufrgs.br/reamat/ CalculoNumerico/livro-oct/in-regras\_compostas.html. Accessed on May 18, 2021.

Babaei, Behzad, et al. 2015. “Efficient and optimized identification of generalized Maxwell viscoelastic relaxation spectra”. Available in: http://dx.doi.org/10.1016/j.jmbbm.2015.10.008. Accessed on May 6, 2021.

Da Cruz, A. M.C. B., Martins, N., Torres, D. F.M. 2012. “Symmetric differentiation on time scales”. DOI:10.1016/j.aml.2012.09.005

Wagner, B., et al. 2021. “A tour of the C# language”. Microsoft. Available in: https://docs.microsoft.com/en-us/dotnet/csharp/tour-of-csharp/. Accessed on: June 14, 2021.

TARECO, M. A. C. 2014. Conceitos de viscoelasticidade na modelação da fluência em estruturas mistas aço-betão. 154f. Dissertação (Mestrado) – Engenharia Civil, Faculdade de Ciências e Tecnologia. Lisboa, 2014. Available in: https://run.unl.pt/bitstream/10362/12481/1/Tareco\_2014.pdf. Accessed on May 4, 2021.

QUEIROZ, José Aparecido Silva de. 2008. “Flexible structures analysis with viscoelastic materials application”. Universidade Estadual Paulista, Faculdade de Engenharia de Bauru, 2008. Available in: https://repositorioslatinoamericanos.uchile.cl/handle/2250/2568006. Accessed on May 4, 2021.

ROSSETTO, N. P. 2009. “Viscosity in stretching tendons”. Universidade Estadual de Campinas. Campinas.

BERNARDES, C., et al. 2005. “Biomechanical parameters' determination for knee joint modeling”. Laboratório de Pesquisa do Exercício, Universidade Federal do Rio Grande do Sul. Porto Alegre.

ZHENG, N.; et al. 1998. “An analytical model of knee for estimation of internal forces during exercise”. American Sports Medicine Institute. Birmingham, Alabama, USA.

1. Responsibility notice

The authors are the only responsible for the printed material included in this paper.